Varianta 35

Subiectul I

a) $AB=\sqrt{13}$.
b) $S_{ABC}=3$.
c) $\sin^2 45^\circ+\cos^2 45^\circ=1$.
d) $\overline{3+4i}=3-4i$.
e) $S=16$.
f) $a=10$.

Subiectul II

1.
a) $m_a=17$.
b) $\frac{3}{100}\cdot 60=1{,}8$.
c) $p=\frac{5}{10}=\frac{1}{2}$.
d) $\log_4\frac{1}{16}=-2$.
e) $r=4$.

2.
a) $f(1)=0$.
b) $x=1$.
c) $f'(x)=3x^2+1$.
d) $\int_0^1 f(x)dx=-\frac{5}{4}$.
e) $\lim_{x\to\infty}\frac{f(x)}{x^3}=1$.

Subiectul III

a) $\det A=1$.
b) $A^2=\begin{pmatrix}1 & 0\\ 6 & 1\end{pmatrix}$.
c) $A^2-2A+I_2=O_2$.
d) Deoarece $A\cdot I_2=I_2\cdot A$ rezultă $(A-I_2)^2=A^2-2A+I_2\overset{c)}{=}O_2$.

e) Pentru $n=1 \Rightarrow A=\begin{pmatrix}1 & 0\\ 3 & 1\end{pmatrix}$ - adevărat.

Presupunem adevărat pentru $n=k$ şi vom demonstra că este adevărat şi pentru $n=k+1$.

Cum $A^{k+1}=A^k\cdot A=\begin{pmatrix}1 & 0\\ 3k & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ 3 & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 3(k+1) & 1\end{pmatrix} \Rightarrow A^n=\begin{pmatrix}1 & 0\\ 3n & 1\end{pmatrix} \forall n\in N^*$.

f) $\det(A)+\det(A^2)+\ldots+\det(A^{2007})=\begin{vmatrix}1 & 0\\ 3 & 1\end{vmatrix}+\begin{vmatrix}1 & 0\\ 3\cdot 2 & 1\end{vmatrix}+\ldots+\begin{vmatrix}1 & 0\\ 3\cdot 2007 & 1\end{vmatrix}=$

$=\underbrace{1+1+\ldots+1}_{2007}=2007$.

g) $A+A^2+\ldots+A^{2007}=\begin{pmatrix}1 & 0\\ 3 & 1\end{pmatrix}+\begin{pmatrix}1 & 0\\ 3\cdot 2 & 1\end{pmatrix}+\ldots+\begin{pmatrix}1 & 0\\ 3\cdot 2007 & 1\end{pmatrix}=$

$=\begin{pmatrix}2007 & 0\\ 3(1+2+\ldots+2007) & 2007\end{pmatrix} \Rightarrow \det(A+A^2+\ldots+A^{2007})=2007^2$.

Subiect IV

a) $f(1)=\dfrac{1}{1\cdot 3}=\dfrac{1}{3}$.

b) $\dfrac{1}{2x}-\dfrac{1}{2(x+2)}=\dfrac{x+2-x}{2x(x+2)}=\dfrac{1}{x(x+2)}=f(x), \forall x>0$.

c) $f'(x)=-\dfrac{1}{2x^2}+\dfrac{1}{2(x+2)^2}=\dfrac{-2(x+1)}{x^2(x+2)^2}$.

d) $f'(x)<0, \forall x>0$, deci funcția f este descrescătoare pe $(0,\infty)$.

e) $a_n=f(1)+f(3)+\ldots+f(2n+1)=$

$=\dfrac{1}{2\cdot 1}-\dfrac{1}{2\cdot 3}+\dfrac{1}{2\cdot 3}-\dfrac{1}{2\cdot 5}+\ldots+\dfrac{1}{2\cdot(2n+1)}-\dfrac{1}{2\cdot(2n+3)}=\dfrac{1}{2}-\dfrac{1}{2(2n+3)}=$

$=\dfrac{n+1}{2n+3}, \forall n\in \mathbf{N}^*$.

f) $\lim\limits_{n\to\infty} n\cdot\left(\dfrac{n+1}{2n+3}-\dfrac{1}{2}\right)=\lim\limits_{n\to\infty}\dfrac{-n}{2(2n+3)}=-\dfrac{1}{4}$.

g) $\lim\limits_{n\to\infty}\int_n^{n+1} f(x)dx=\lim\limits_{n\to\infty}\int_n^{n+1}\left(\dfrac{1}{2x}-\dfrac{1}{2x+4}\right)dx=\lim\limits_{n\to\infty}\dfrac{1}{2}\left(\ln\dfrac{n+1}{n}-\ln\dfrac{n+3}{n+2}\right)=0$.